Appendix I: Traps

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- times any number gives you zero 4
- on top of a division problem also gives you zero. 5
- on the bottom of a division is the only problem no one, and no calculator can do 5

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3-Level Ratios ***6***
- work just like regular ratios. They are used in geometry, and in many problems involving totals, and percents of totals. 6

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4. Put the information you have on the top. (Don't be afraid to sum up some scores). If you're looking for the last test put one of the X's up on top. 11

1, The same. 12

2. Be careful here. We have three test scores, but know that there will be another test, because what we are looking for is the grade she must get on that test 12
3. Since we know the average that we want to get, put it down. It may look strange to have something other than the ? or the x after the equals

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- When you change a decimal to a percent, you ALWAYS move the decimal point *two* places to the *right.* 33

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0 and 1 Trap

Zero —

- plus any number doesn't change the number at all.
- times any number gives you zero.

You if you find that $x = 0$, even a weird-looking problem like

$x^3 - 5x^2 + 5x = b + 19$ reduces down to

$$0 = b \cancel{+ 19}$$

$$\underline{-19 \quad\quad} -19$$

$$-19 = b$$

- on top of a division problem also gives you zero.

If ¼ means "one quarter", $^0/_4$ means "no quarters". Pretty normal, unlike . .

- on the bottom of a division is the only problem no one, and no calculator can do.

Try it, and you'll get an error.

So, you can just remember the rule, or read on.

One was to think of the bottom number is "How many pieces." One pizza is divided into eight pieces, so one slice is ⅛-th of a pizza.

OK, so an older person comes along and says "Oh I shouldn't have a whole piece, so Ill just get a half". So they cut a slice in half. (That's $^1/_{16}$ if you check out multiplying fractions). So you can divide anything into smaller and smaller pieces.

If you just kept going you'd end up with molecules, divide them into atoms, and then divide the atoms in to protons, neutrons, electrons. It's impossible to divide anything down to zero in nature too.

One —

When you multiply anything by one, it doesn't change. Duh. This fact is so simple that it leads to a potential problem:

- When simplifying fractions, you're really multiplying the original fraction by one.

In problems like ½ + ¼ ,you know that you have to change the one-half to two fourths. What you're really doing when you convert one-half to two-fourths is using two rules:

1. multiplying by one gives you the same number
2. a number over (or divided by) itself equals one

$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$ To Start: you had to multiply by 2 in order to get four on the bottom Many teachers suggest writing the number multiply by, so

you'll remember to
(Rule 2) keep the same number on the top and bottom. That was, you'll . . .
(Rule 1) be multiplying the original fraction by one. We may have a four on the bottom, but one half is really the same as two fourths.

3-Level Ratios

- work just like regular ratios. They are used in geometry, and in many problems involving totals, and percents of totals.

Using Triangles

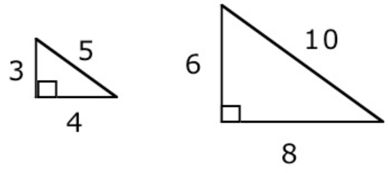
We'll use "similar triangles" for our example. these triangles have the same shape, and therefore the same angles. One is just bigger or smaller than the other.

You can imagine zooming in and zooming out of a computer screen. A triangle on the screen would just get bigger or smaller, but not change its shape. That's all.

A special kind of triangle which you see on many tests is called a 3-4-5 triangle.

It's a "right triangle", which means it has a 90° angle (The little square in the drawing) joining the two shorter sides.

The longer side is called the hypotenuse, which we can just label "h"."



Triangle A Triangle B

short side	3	6
long side	4	8
h	5	10

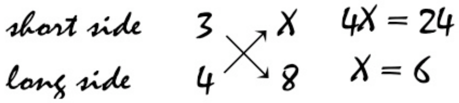
Note that you can take each side of Triangle A, multiply it by 2, and get to Triangle B.

$$\begin{array}{cccc} & \frac{1}{4} & & \frac{1}{4} \\ + & \frac{1}{2} & \times\frac{2}{2} & \frac{2}{4} \\ & \underline{\quad} & & \underline{\quad} \end{array}$$

This is the exact same procedure you learned in grade school when you multiplied both the top and bottom of the ½ in order to change it to $^{2}/_{4}$. The only difference here is that you do the same thing on three rows instead of two.

But what about cross multiplying, when there are strange numbers and you can’t pick a number like 2?

You can pick any two rows and cross multiply them:



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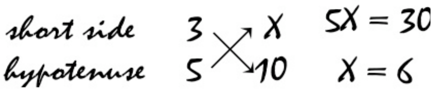
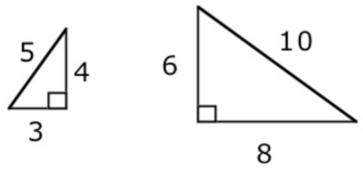


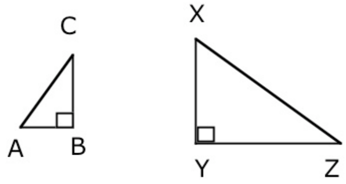
In a three-level ratio, it doesn’t matter which two rows you pick to cross multiply.

Where you might run into problems is setting up the initial ratio. That’s why, just like in other ratios, it helps to organize the numbers by writing down what they are first, like “long side” or “short side”.



Triangle A Triangle B

or



Triangle A Triangle B

Area Problems

Area is always expressed in “square” units: square inches, square meters, square feet; sometimes written in^2, $meters^2$, ft^2. The ^{2} helps people remember these are “two dimensional” or “2D” problems: each figure has two measurements. They’re usually called length and width.

The formula is so obvious people can miss it. How many square inches are in a “three by five” card? Fifteen, right? What does it usually say on the label: 3 ____ 5.

How about an “eight by ten” rug, or room. That’s written 8 ____ 10. If you put an X in the blanks you’re right, and that’s the formula.

An easy way to remember the formula for squares and rectangles: Just imagine you’re asking “How many square-foot tiles do I need to cover this area?"

Test Hint:	• Many tests give you problems with strange stapes, and ask you to find the area. What you need to do here is: 1. Break the problem down into simpler shapes, like circles,

	squares, and triangles, for which you can use familiar formulas to find the area. 2. Be sure to list the areas of *all* the simpler shapes. (There's an example in the **Carrying Errors** section.) 3. Add or subtract as the problem requires.

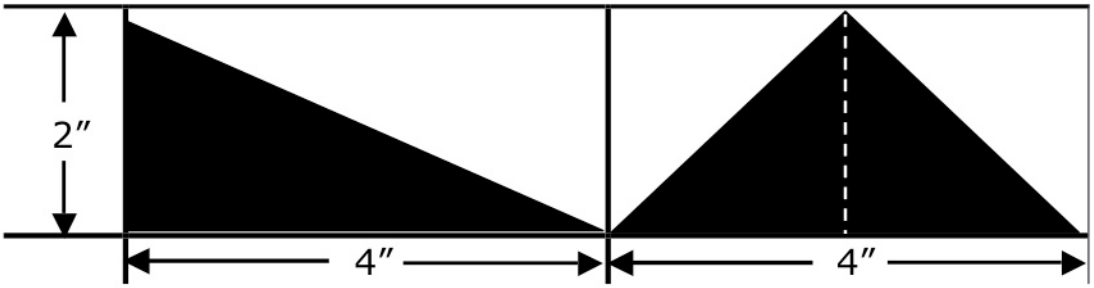
What throws some people is that we still use "square inches" for triangles, circles, and other odd shapes which aren't squares. So we have some "special formulas:"

Triangle Rule:

- Find the area around of the box around the triangle, and cut it in half.

You probably remember that, for triangles,

Area = (Base * Height) ÷ 2. This rule works because *any* rectangle can be cut into two triangles:



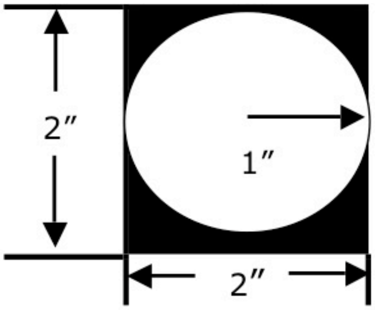
It's fairly easy to see that the rectangle on the right is 2″ x 4″, or 8″, so the area of the triangle has to be 4 square inches.

It's not as easy to see that the same rule apples to the triangle on the right, but if you drag the corners of a triangle on a computer, or cut triangles out of rectangular pieces of paper and measure them, you'll see that the rule *always* holds true.

When most math books talk about "base times height", they mean that the four inch bottom is the base, and the two inch side (which they usually draw as a dotted line running up the middle of the triangle) is the height.

Circle Rule #1:

- The area of a circle is πr^2. Here's what that means:



- We know that π is 3.14
- We know that 2″ is the diameter, so half the diameter is the radius (r). Or 1″.
- One squared is 1. So
- $\pi * 1 = \pi$. So the area of the circle is 3.14 in^2, or π in^2 if you're into math.

So we can see why pi is close to 4: the circle just about fills the square. The shaded portion is the difference between four and pi. (Area of the Square – Area of the Circle)

After all this explanation, you might just want to remember Area = $\pi * r^{2.}$ Most tests give you this formula There's just one little difficulty. Pi is used in *two* formulas about circles. We just did the one for Area, but there is also one for the "circumference".

"Circumference" is a special term for the perimeter of a circle. (If you're not sure about perimeter, you can think of a fence, which goes around the edge of a property; or a war game where the captain says 'Guard the Perimeter'.) There are actually two formulas for perimeter: one is good, the other evil:

Circle Rule #2— Good and Evil Versions

- The evil formula: circumference = 2 * π * radius ($c = 2\pi r$)
- The good formula: circumference = diameter * π ($c = d\pi$)

The problem with the "evil" formula is that is looks so much like the formula for area: $2\pi r$ vs. πr^2. How many students suffered through tests wondering "Is this the formula with the two up top or not?"

Why not just teach that $c = d\pi$? There's really no good answer. Somebody probably thought it would be clever to take the diameter, which is twice the radius, and call it 2 * r. Then he multiplied that by π.

Since the variables in an equation are always in alphabetical order (with numbers first.), we ended up with $c = 2\pi r$.

Now, if you want a little more clarification on the area vs. the circumference:

1. Area is always measured in square (somethings): square inches (in^2), square feet (ft^2) or yards (yd^2), square

meters (m^2) or square kilometers (km^2). So you know to use the one on the top.

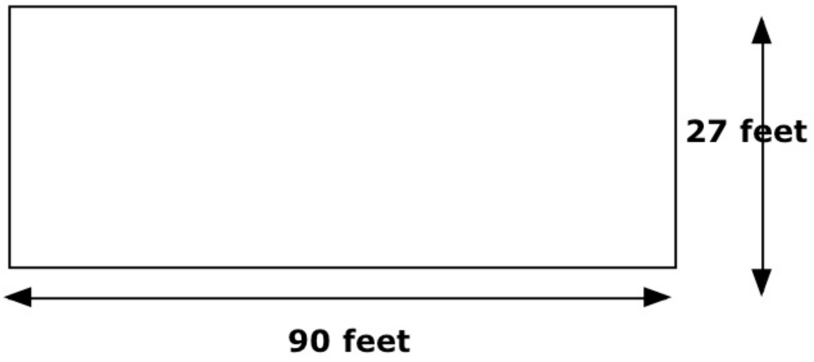
Area is two-dimensional, like Pac Man and the original Donkey Kong. "Dimension" just means "measurement", so we can see that area has two measurements: a 10 foot x 10 foot room, for example, needs 100 square-foot tiles.

2. Perimeter is one-dimensional. If we were putting baseboards around the 10 x 10 room, or molding around the ceiling, we'd need to buy 40 feet of it. (not just square feet.) So there are no square measurements in perimeter or circumference.

Watch out for square feet vs. square yards

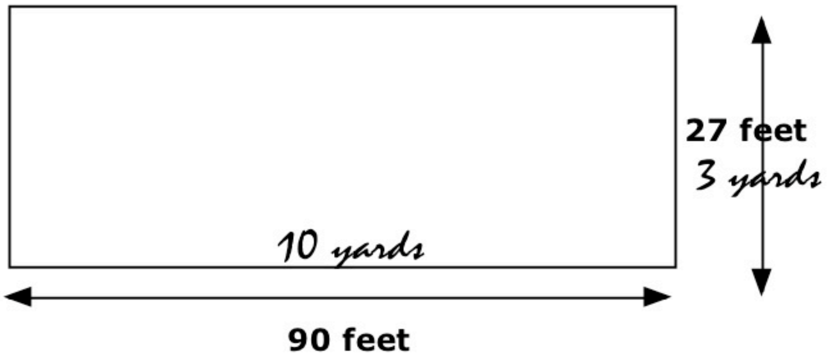
While most floor plans are laid out in square feet, some things like carpet or lawns are measured in square yards. But you can't just multiply or divide square feet by three because there are three feet in a yard.

Here's a lawn. How many square yards are in it?



You can multiply 90 x 27 to get 2,430 square feet. You know you can divide six feet by two to get three yards. But 2,430 ÷ 3 = 810, and this looks a little too big.

Let's try another tactic: we'll change the feet into yards on the drawing:



We can see that we’ve got 30 square yards, which looks reasonable.

Here’s the trap: If you look at a square yard, it’s got three feet on each side. So, 3 x 3 is 9. There are 9 square feet in one square yard.

If you’re ever not sure about the formula for a conversion, you can do a really simple one to get it.

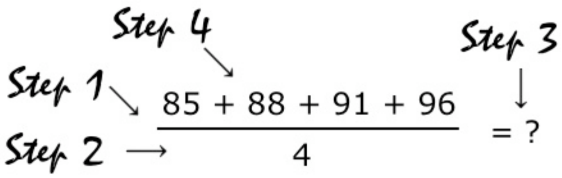
Averages

- Also called the **Mean**.

Averages are so common—in sports, grades, prices—that it’s difficult to define them. So, we’ll go right to the procedure. with two grade-type examples. If you’re pretty good on averages, but came on a strange-looking problem, you might want to skip to the second one.

① Scott’s four test scores are 85, 88, 91, and 96. What is his grade so far?

Here is the procedure for *all* average problems:



1. Draw the bar
2. Write total number of tests/things to average. Put it on the bottom
3. Draw the equals sign. If you don’t know the average, as in ①, put a question mark. If you know the average that you want to get, put the average down. The question mark will probably appear in Step four.
4. Put the information you have on the top. (Don’t be afraid to sum up some scores). If you’re looking for the last test put one of the X’s up on top.
5. Then just solve it like a normal equation. First, add up the individual tests. The total is 360. Divide that by 4, and the answer is 90.

② Amanda’s last three tests scores were 85, 88, and 91. She needs to get a 90 average. What score must she get on the next test?

The numbers look very similar to the ones in last problem, and we can follow the same steps:

Step 4 ↘ Step 3 ↓

$$\text{Step 2} \longrightarrow \frac{85 + 88 + 91 + ?}{4} = 90$$

1, The same.
2. Be careful here. We have three test scores, but know that there will be another test, because what we are looking for is the grade she must get on that test.
3. Since we know the average that we want to get, put it down. It may look strange to have something other than the ? or the x after the equals sign, but sticking to procedure is what gets us through different-looking problems.
4. Put the information you have on the top. Since we're looking for the last test, we'll put the ? here
5. Here's how to solve this equation:

   a. We have to get rid of the four first. If not. we'd have to divide each of the tests, and the ?, by four. We can multiply both sides of the equation by 4. This way, the 4 on the left cancels out.

↙ The two fours on this side cancel

$$\frac{\cancel{4} \times (85 + 88 + 91 + ?)}{\cancel{4}} = 90 \times 4$$

↙ Add up the three grades we already have

$$264 + ? = 360$$

So we're left with a plain subtraction, which gives us 94, same is in the last problem.

Carrying Errors

Fractions

Remember when teachers would talk about "borrowing" from the tens column? Maybe changing a dime into ten pennies. (This was much easier when dimes could actually buy things. Maybe teachers should start talking about $10 and $1 bills.) You probably got so good at this carrying that it was a habit. So good that you forgot how you learned it. All you need to do now is just think back to when you just need a few changes for certain types of problems

If a pizza shop has one normal eight-slice pizza $\boxed{\text{and}}$ two slices, how many $\boxed{\text{slices will they have left}}$ after they sell 5?

Step 1: Ok, we've underlined the facts and circled keywords and questions. Now, let's look at the first clause (up to the comma):

1 One pizza is eight slices,
2. Add we have two additional slices
3. So we've got (one times eight) + 2 = 10 slices

Let's think of the one pizza as $^{8}/_{8}$ slices, and answer the question How many slices (eights) to we have?

$$\frac{2}{8} = ?$$

ⓐ Start at the Bottom (eights)

ⓑ Go around CLOCKWISE. Eight times one is eight. (This sounds too simple, I know. It's easier to think of two pizzas: Eight slices each, times two, is 16 slices)

ⓒ After Eight times one is eight, add two. so we've got ten slices.

Step 2:

$$\frac{10}{8} - \frac{5}{8}$$

ⓐ Since we wrote down the $^{10}/_{8}$ for Step 1, now all we need to do is subtract ⅝ .

So We've got five slices.

Remember, with adding and subtracting pieces—or fractions—the size of the part doesn't change. One slice is still ⅛ of a pizza.

Non Metric Measurements

It usually easier to do the problem in the *smallest* units: inches, not feet, for ex. Let's start with a really simple example.

① You've got a two-foot two-by-four, but need one 18 inches long. How much you need to cut off?

ⓐ Convert 2 feet to 24 inches.

ⓑ Now you just multiply.

From this example, you can see that when you go from a *bigger* unit (feet) to a *smaller* unit (inches) you multiply. This rule is *always* true.

② How many six-ounce cups can you fill from a one-gallon jug of lemonade?

ⓐ Convert one gallon to 64 ounces.

ⓑ Since you are "dividing" the gallon of punch among a bunch of six-ounce glasses, the equation is $64 \div 6 = x$. The answer is 10.66667, so you have to decide what to do about the remainder. Since the problem asks how many glasses you can "fill", and you're probably not going to chop up a paper cup, 10 is the right answer.

Time

"*Elapsed Time*" means how much time has passed. One thing to remember when setting up these problems:

Rule:

- Since we can't go back in time, the answer is never negative. So *the later number* is *always on the top.*

① First Period starts at 7:30 and goes until 8:15. How long is it?

8:15 ← Put the later time on top.

<u>- 7:30</u> ← line up the colons in a straight column, just like you'd line up any number

Rule:

- In time problems, the minutes act like <u>a single column</u> in a more normal problem.

So, we need to "borrow" an hour, and put sixty *more* minutes in the minutes column.

It's important to remember if there are already minutes, we need to *add the 60 minutes we borrowed.* (Not just change the minutes column to 60.

7:75 ← 60 + 15 = 75

~~8~~:~~15~~

<u>- 7:30</u>

45 min.

When you are dealing with afternoon times, it's easier to use "military time" or the "24-hour clock". The purpose of this clock is to get rid of the "a.m." and "p.m." So if noon is 12 o'clock, the next hour can't be one, since there's already a one a.m.

The next hour *has* to be 13 o'clock, since nothing else would make sense. So if you see an hour greater than 12, it has to be the afternoon. (Fortunately, the minutes don't have to change.)

Since the military can't have any confusion about time, they say things like "Oh eight hundred hours", which just means 8 a.m. Here's how to pronounce military time:

Rule:

- Military, or 24-hour time, does not use AM and PM.
- For afternoon times, add twelve to the regular p.m. hour.
- All times before ten a.m. have an "Oh" before the hour.
- All hours have "hundred" after them, instead of "o'clock".

So 5 a.m. is "Oh five hundred hours"; 9 a.m. is "Oh nine hundred hours". Ten a.m. is "ten hundred hours". One p.m. is "thirteen hundred hours".

But why do they say "hundred" ? Especially since the minutes *don't* change in military time?

Because they need two spaces for the minutes. 8:30 a.m., for example, is pronounced "Oh eight thirty" and written "08:30". Five-forty-five in the morning is pronounced "Oh five forty-five" and written "05:45".

Rule:
- In military time, the minutes *do not* change.
- Military time *always* allows two places for the hour and two places for the minutes.

Just as you put a zero in the tens place when you say "two hundred and five", you fill in zeros so the 24-hour or military clock *always* has four places.

Even if you never join the military, you will often see 24-hour time in business applications like timecards.

② Joe started work at 7:30 am and left at 2:30 p.m. How many hours did he work?

14:30 ← 4:30 p.m. in regular time.
-07:30
7:00 hours ← subtract minutes and hours just as in a traditional problem.

③ Alisha started work at 7:30 am and left at 3:00 p.m. How many hours did she work?

13 : 60 ← Borrow one hour from the hours column
~~14~~:~~00~~ and add 60 to the minutes column
-07:30
7:30 ← Seven and a half hours.

④ Maria started work at 7:30 am and left at 3:15 p.m. How many hours did she work?

13 : 75 ← Yes, this is the same problem as ③, but
~~14~~:~~15~~ remember we have to add the 60 minutes
-07:30 we borrow *and* the 15 minutes already there.
7:45

Change, percent going up or down

- Remember that these are basically ratio problems. What can throw some people is if the percent is going up or down. There's a Merchant Problems section with examples involving $: discounts, tips, sales tax, commissions, wholesale costs and profits. This section deals with other things, like the population of a school, going up and down.

Distributive Property

- In some examples, we call this the "Recipe Property" or the "Oil Change Property"

① Alisha is planning a party. She figures that she needs two burgers and three sodas for each guest. If 12 people are coming, how many sodas does she need?

The two burgers and three sodas are a kind of set — she needs one set for each person. There's no way to "add" burgers and sodas, so we can just write the set (2*Burgers + 3*Sodas) . We'll put the times sign in to make it a little more like the math sentence (2B and 3S).

Now you know that for twelve people Alisha needs 24 burgers and 36 sodas. But how do you know it?

Well, you multiplied the 2 burgers by the twelve guests, and then you did the same thing for the sodas. Three sodas for (times) twelve people is thirty-six sodas. You know how to do this in English already.

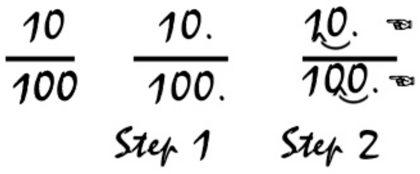
Here's the same thing in Math: $a (x + y) = ax + ay$. You "distribute" the a across the $(x + y)$.

If you think of the variables in the parenthesis as a set of things, you won't have much trouble with this property.

Dividing Decimals

- If the decimal problem looks like a fraction, you can *divide* my moving the decimal point to the *left*—as long as you move the top and bottom the same number of spaces.

Remember that to get a number that's **L**ess, you go **L**eft—just like the < **L**ess than sign.



A dime as $^{10}/_{100}$, or $^{1}/_{10}$ of a dollar, as we know. Let's see how we got there.

Step 1: We know that a dime is10 out of 100 cents. What people might forget and that every whole number has a period after it, whether it's written down or not. So . . .

Step 2: We can divide both side of the fraction by 10, when we move the decimal to the left. (Since division makes a number less, we know to go left.) You can use the ^ sign to remember which of the periods is the "new" one.

- Changing a number to a percent is really putting it over 100. So . . .
  - We're dividing the number by 100.
  - One hundred has two zeros so we go over two places. (You can see Scientific Notation for more information on this procedure.)
  - Since the number will be less, we go left. (Like the < sign)

The easiest example is one-half, since we know this is 50%:

② What is 50% of 100?

1. Start with 50%. The two zeros in the % sign remind us that we're really changing it to $^{50}/_{100}$.

2. 

Start with the point which, remember, is behind ever whole number. Move it two places left.

3. "Of " is always times. So .50 x 100 = 50, which we know. but it's a good way to check the procedure.

③ Sandi lives in a state with a 5% sales tax. What is the tax on $100?

1. We're doing $^{5}/_{100}$ rather than $^{50}/_{100,}$ the same thing is dividing 5 by 100. There's a little extra step here, which tricks some people up:

2. 

We add the zero after the 5, as usual, and go left. But we can't put the period in front of the five.

3. 

Just like we can put any number of zeros *after* a number, we can put any number of zeros *before* a number too. Here, we just need to add one zero, so we can go over two places.

4. So, .05 x 100 = 5, or $5 in tax.

5. One good thing about math is that the same rules *always* work. No "I before e except after c except in receive" rules here. A percent to a decimal is *always* "move the decimal two places to the left".

Using Long Division with Decimals

- If there is a decimal in the first number, move it back until you have a whole number.
- Make sure you move the decimal in the second number, the one under the √ sign.

Keeping with our rule that we should use a simple example:

① Rashid has a $10 bill. How many quarters can he get for it?

We know that he's *dividing* the bill into quarters. There are easier ways to figure out he'll get 40 quarters, but, to check our rules:

1. $^{10}/_{.25}$ is the same as $.25\overline{)\,10}$

In order to get the .25 to be a whole number, we need to move the decimal two places to the *right.* No matter which form we use, as long as we do the same thing to both numbers we're OK.

- in fraction form:

$$\frac{10.\overset{\frown}{00}{}_\wedge}{.\overset{\frown}{25}{}_\wedge}$$

↖ There is always a . behind any whole number, which we can move two places to the *right.*

- in long-division form:

$$\begin{array}{r} 40_\wedge \\ .\overset{\frown}{25}{}_\wedge\overline{)\,10.\overset{\frown}{00}{}_\wedge} \\ \underline{100}\downarrow \\ 0 \end{array}$$

To make a number greater, move the decimal to the right. Just like the greater-than sign, >, points to the right.

You can see that to MULIPTLY a number, you move the decimal to the RIGHT.

Estimating

- for a "range" of values, such as a tip which is between 15% and 20%, see Merchant Problems , Tips.

When you know what "place" you must estimate to.

You do this so often it's automatic. Let's say you live in a state with a 5% sales tax, and you want to find the tax on $10.50. Multiply .05 x 10.50 and you get $\boxed{0.525}$. So we've got 52½ cents. But wait, where are you going to get a half of a penny. (The U.S. discontinued that coin in the 1800's

FIX—half penny).

When you round money, you round to the nearest cent, which is really the "hundredths place". You know you'll end up paying 53¢ tax.

Here's the procedure:

1. Go to the place <u>after</u> (or to the right of) the one you're looking for. (In our example, it's the "thousandths" place

Over or Under Estimating

Suppose we have to estimate a price for carpeting a room. Here is the floor plan. Having too little carpet would be a disaster, so we will round our estimates up.

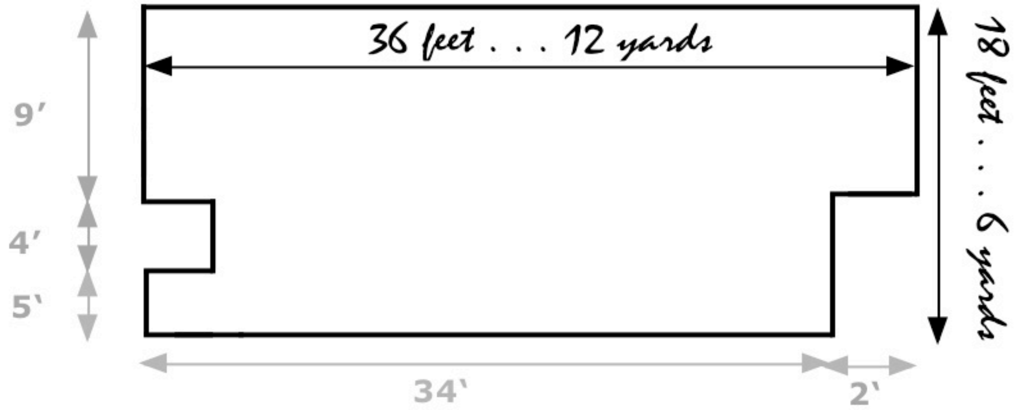
In many cases, you'll want to work in smaller units. To work in feet here, please remember that you need to convert *nine* square feet to one square yard. (There's an explanation in the **Area** section if you're not sure why.) but because you're going to be pricing the job and buying the materials in square yards, it's easier to estimate that way.

Rule:

- When estimating the amount of material you will use in construction and manufacturing, you always round up.

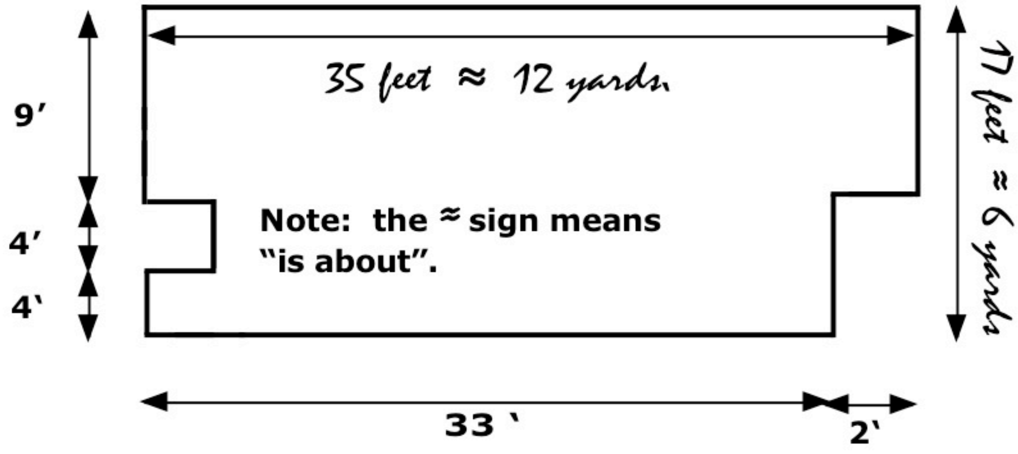
Here's why: whether you're building a house or making a craft project, you usually start with something—a piece of wood, metal, even stone—and then take some away.

That is why you can just ignore the odd-shaped cut-outs. It doesn't really matter when you're buying carpet that some of it will be scrapped; you still have to buy the rolls, and charge the customer accordingly:



Twelve by nine yards is 72 yards. Or, you can multiply 36 by 18 to get 648 square feet, and then divide by 9, to get the same result.

Even if we assume that the room doesn’t have sizes which convert exactly from feet to yards, we can use the rule that in construction you start with a bigger size. Here’ it’s a yard, not a fraction of a yard:



Note: the ≈ sign means “is about”.

Formulas, changing them

- Look under **Variables** , **But Why Do All This?** for an example of changing around formulas.

Fractions

Three things make Fractions complicated:

- Two sets of rules: one for + and –; another for x and ÷.
- \+ and - are *harder* than other operations
- *Multiplying* fractions makes them *smaller* ,while *Dividing* fractions makes them *bigger.*

1. What confuses most people about fractions is that there are two sets of rules: You can think of Addition and Subtraction as operation (since they're just backwards versions of each other: remember those 'fact families' in elementary school?), and Multiplication and Division as another. Fractions have different rules for each operation.

Adding and Subtracting Fractions

2. One thing that's backward about fractions is that addition and subtraction are harder than multiplication and division. Back in the day you could add on your fingers if you had to, but you had to memorize the times tables. Fractions are just the opposite: adding and subtracting is hard because when you're adding pieces you have to talk about the same thing. Let's take a standard eight-hour workday, and assume that we're worked an hour and a half on Monday, two full days Tuesday and Wednesday, and one-half of Thursday.

The easiest way to add up the time is to use the smallest unit of time, the hour:

		hours
Monday	1 ½ hours	1 ½
Tuesday	1 day	8
Wednesday	1 day	8
+ Thursday	½ day	4
Total		21 ½

You can see that we had to use hours, which are really pieces of the workday as the units we use to add up how much time we work.

In this example, we used the smallest unit of time which we could. Most measurements work the same way. Eggs, for a simple example, are usually sold by the dozen. But if someone's making omelets, they say "3 eggs", not "¼ dozen". At a pizza stand, people don't say "I'll take a quarter of a pizza" they say "two slices".

When you first learned fractions, you probably had an example like "What is one-half plus one fourth." The teacher probably had you write them in a column (not straight across), and then write the equivalent fraction, next to it, like this:

$\frac{1}{2} \quad \frac{2}{4}$ ← As long as we multiply both the top and bottom by the same number, we're OK. (see "1 and 0" traps)

$+\ \frac{1}{4} \quad \frac{1}{4}$ ← since 4 is the bigger number (and two goes into it evenly), we'll use 4 as the bottom number. You can also say "We'll do the problem in fourths"

$\frac{3}{4}$ ← Getting the bottom number (the denominator) to be the same ("common", here, just means "the same") is the hard part.

Here are a couple of rules for finding a "common denominator":

- If you can see a number that both numbers go into, you can use that number.

The problem just above is a good example of this procedure. If you don't remember about multiplying both the top and bottom by the same number, you can skip ahead, over the next section.

- If you're not allowed to use a calculator, you need to "factor" the denominators. You can to the Factoring Section for some help. If you can use the calculator, you can skip to the next rule

- With a calculator, the easiest way to get a common denominators is to just multiply the two denominators together.